Verification – Radial flow to a well

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Introduction

Aquifer testing can be conducted on hydraulic wells to determine hydraulic characteristics of an aquifer, as well as for predicting the drawdown of the piezometric surfaces within an aquifer resulting from the pumping of water from a well. There are a few different analytical solutions that can be used to predict the flow of water to a well or to determine the hydraulic characteristics of an aquifer. Each analytical solution is dependent on the conditions existing at the well during the time of testing (Schwartz and Zhang, 2003).

The objective of this example is to use SEEP/W to simulate the drawdown experienced within a confined aquifer to pumping of a well. The Theis solution will be used for predicting the drawdown of the piezometric surface resulting from pumping of water from a well in a horizontal confined aquifer. If the aquifer's hydraulic properties are known, it is possible to predict the drawdown at any distance from the well at any time after the start of pumping. Comparing such a solution with a SEEP/W analysis makes it possible to verify the axisymmetric and transient features of the software.

Background

The Theis solution can be used to evaluate the effect of a pumping well on a confined aquifer. The drawdown (s) is defined by the Theis solution by:

$$h_0 - h = s = \frac{Q}{4\pi T} W(u)$$

Equation 1

where h_0 is the initial hydraulic head, h is the hydraulic head, Q is the pumping rate, T is the transmissivity of the aquifer, and $W(u)$ is the well function, which is expressed as:

$$W(u) = \int_u^\infty \frac{e^{-y}}{y} dy = -0.577216 - \ln(u) + u - \frac{u^2}{2!2} + \frac{u^3}{3!3} - \frac{u^4}{4!4} + \ldots$$

Equation 2

$$u = \frac{r^2 S}{4Tt}$$ Equation 3

where r is the radial distance from the well axis, t is the pumping time and S is the storativity. The Theis solution assumes that the pumping well is full penetrating with a constant pumping rate and negligible storage, that the aquifer is confined, homogeneous and isotropic, and that all water being pumped comes from aquifer storage and is discharged instantaneously (Schwartz and Zhang, 2003).

Numerical Simulation

A sketch of the example problem is shown in Figure 1. The aquifer is 5 m thick (l) and the total hydraulic head in the aquifer is 16 m. The aquifer has a storativity of 0.05 and a transmissivity of 0.01 m^2/sec. The well screen is 0.3 m in diameter (r=0.15 m) and extends over the entire depth of the aquifer. The pumping rate (q) is specified as 0.025 m^3/sec/m^2 (or $q * l$=0.125 m^3/sec). The total flow out of the well (Q) can then be calculated using:

$$Q = q * A = q * l * 2\pi r = 0.025\ m^3/sec/m^2 * 5m * 2\pi * 0.15m = 0.1178\ m^3/sec$$ Equation 4

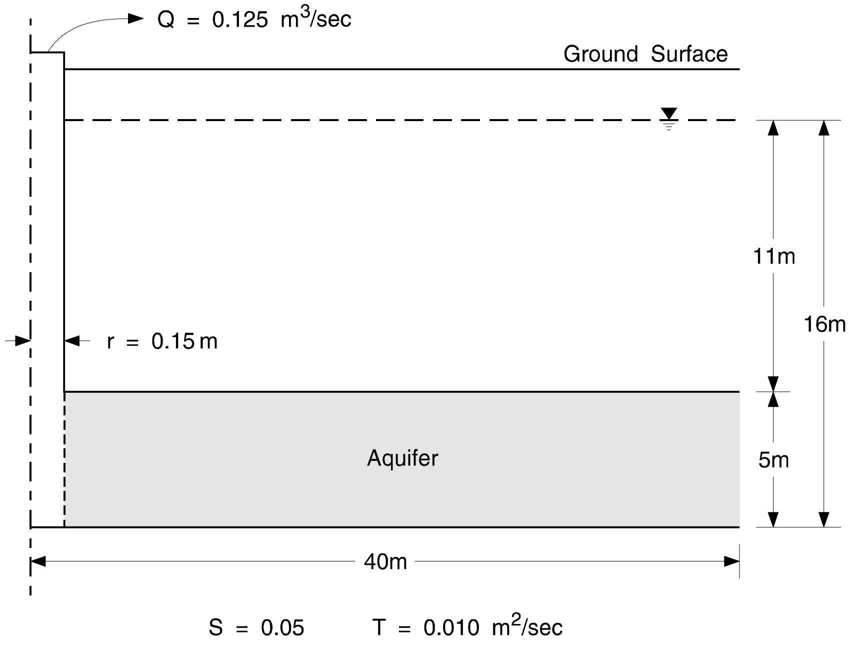


Figure 1. Diagrammatic sketch of problem.

Figure 2 shows the axisymmetric configuration of the example problem in SEEP/W. Since the aquifer is considered to be fully confined, only the aquifer itself it modeled in the analysis. The model domain is a horizontal column that is 5 m in height and 40 m in width. Note, the x coordinates of the line representing the well screen are set to 0.15 m (Figure 3). In an axisymmetric analysis, it is important to include the appropriate radius for the pumping well, in this case, being simulated, as the internal solver will use the distance from the axis (x=0m) to calculate the volume of water entering or leaving the domain along assigned boundary conditions. In the Define Geometry Properties window, the Central Angle can be set to 360° to consider the entire volume of the circular well (Figure 4). If only a section of the well volume is needed, the Central Angle can be set to approximately 57.2958°, changing all resulting flow values to be calculated per radian. Since the entire water volume leaving the well throughout the analysis is desired, the Central Angle will be kept at 360° for this analysis.

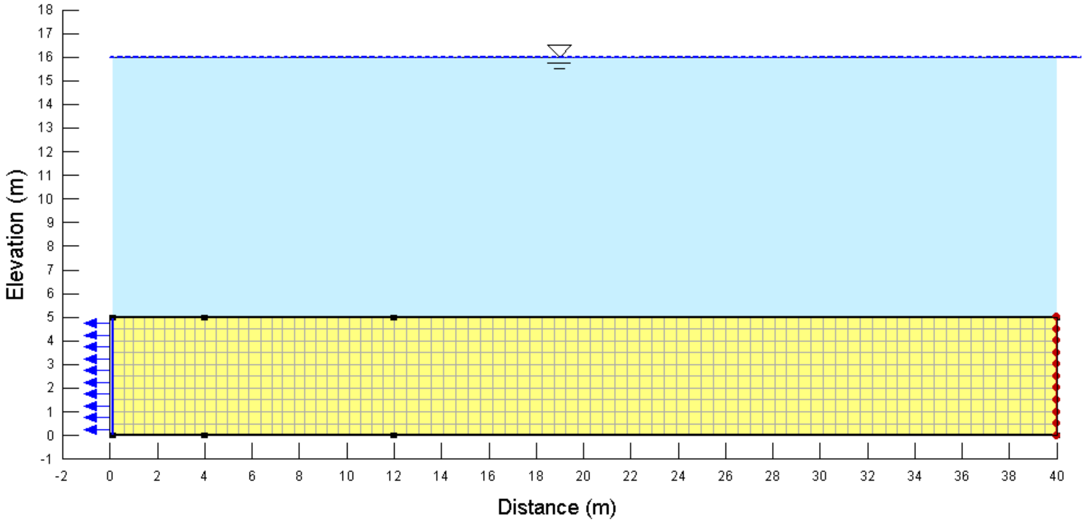
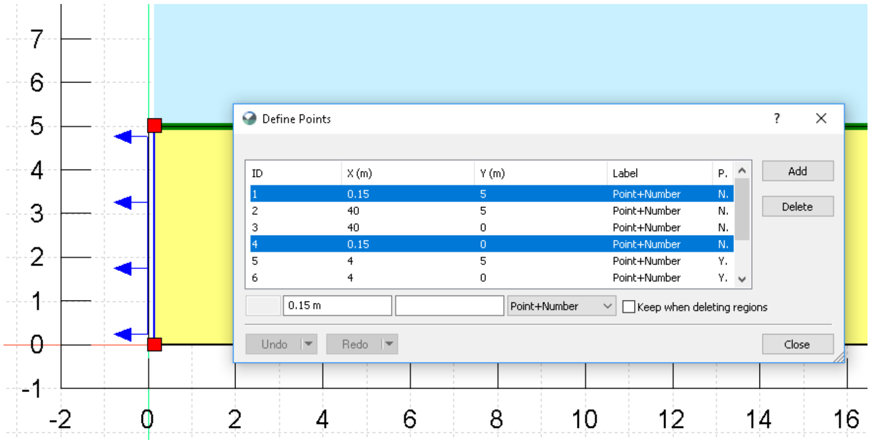


Figure 2. Problem configuration in SEEP/W.



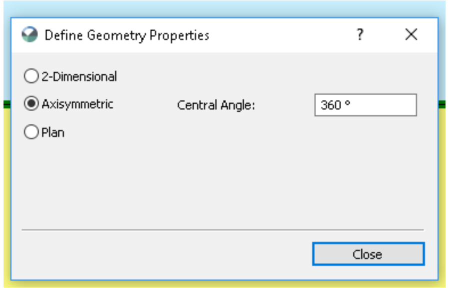
ID	X (m)	Y (m)	Label	P.
1	0.15	5	Point+Number	N.
2	40	5	Point+Number	N.
3	40	0	Point+Number	N.
4	0.15	0	Point+Number	N.
5	4	5	Point+Number	Y.
6	4	0	Point+Number	Y.

0.15 m

Point+Number

☐ Keep when deleting regions

Figure 3. Nodal coordinates of the well screen in an axisymmetric analysis.


Define Geometry Properties

○ 2-Dimensional

● Axisymmetric Central Angle: 360 °

○ Plan

Figure 4. Axisymmetric settings for simulating the entire well volume in the Project Settings window.

The unit flux boundary condition set to -0.025 m/sec was used to define the pumping rate along the line representing the well screen, where the negative sign ensures that the water is being forced to leave the domain. A total head boundary condition of 16 m was set to the far right boundary condition, as this boundary is considered to be far enough away to not influence the results of the aquifer test. This will ensure that the background total head of 16 m experienced by the aquifer is simulated throughout the duration of the analysis. The initial pore-water pressure conditions for the transient analysis were simulated using the initial water table option, with the initial water table set to 16 m for the entire domain. This means that no steady-state analysis is required to define the initial conditions of this analysis.

The saturated only material model is used in this example, as the aquifer will remain saturated throughout the analysis. The saturated volumetric water content was set to 0.4. SEEP/W also required the coefficient of volume compressibility (m_v), which is the slope of the volumetric water content curve (or the soil-moisture characteristic curve), to represent the storativity of a material. The value of m_v (slope of the storage curve where the pore-pressure is positive) corresponding to a storativity (S) of 0.05 can be calculated as:

$$S_s = \frac{S}{b} = \frac{0.05}{5m} = 0.01\ /m$$

Equation 5

$$m_v = \frac{S_s}{\gamma_w} = \frac{0.01\ /m}{9.81\ kN/m^3} = 0.001\ /kPa$$

Equation 6

where S_s is the specific storage, b is the aquifer thickness, and γ_w is the unit weight of water.

SEEP/W uses hydraulic conductivity (K) rather than transmissivity (T). The hydraulic conductivity corresponding to a transmissivity of 0.01 m^2/sec in a 5 m thick aquifer can be calculated as:

$$K = \frac{T}{b} = \frac{0.01\ m^2/sec}{5m} = 0.002\ m/sec$$

Equation 7

The global element size was set to 0.5m in the Draw Mesh Properties window. The transient analysis is set to simulate a total duration of 20 minutes over 10 time steps that increase exponentially from an initial time step of 10 seconds.

Results and Discussion

The resulting pore-water pressure contours and drawdown following 20 minutes of pumping is shown in Figure 5. The simulated drawdown curves for each time step are also shown in Figure 6. To further understand this analysis, a comparison with the Theis solution must be completed. The calculations can be conveniently done in an EXCEL spreadsheet.

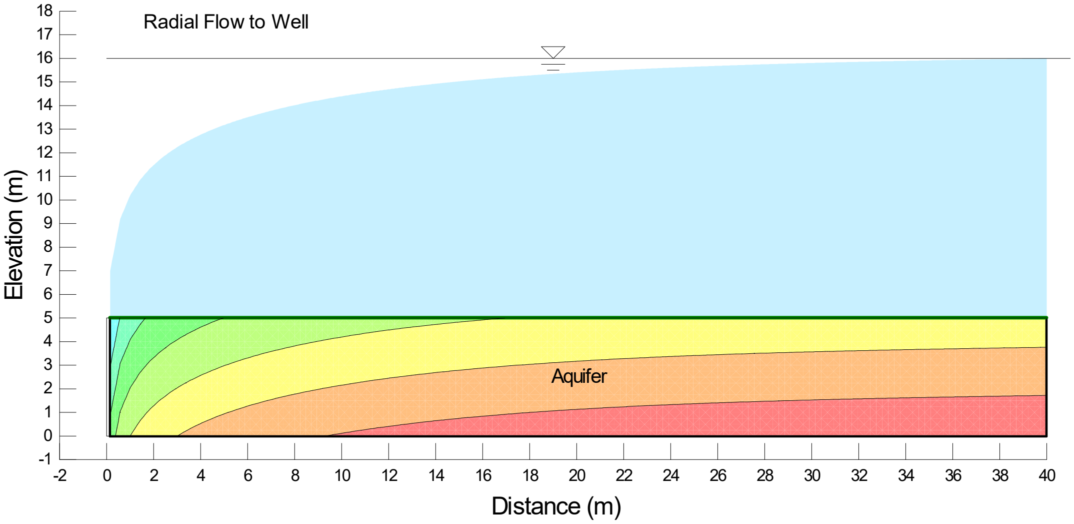


Figure 5. Drawdown after 20 minutes of pumping.

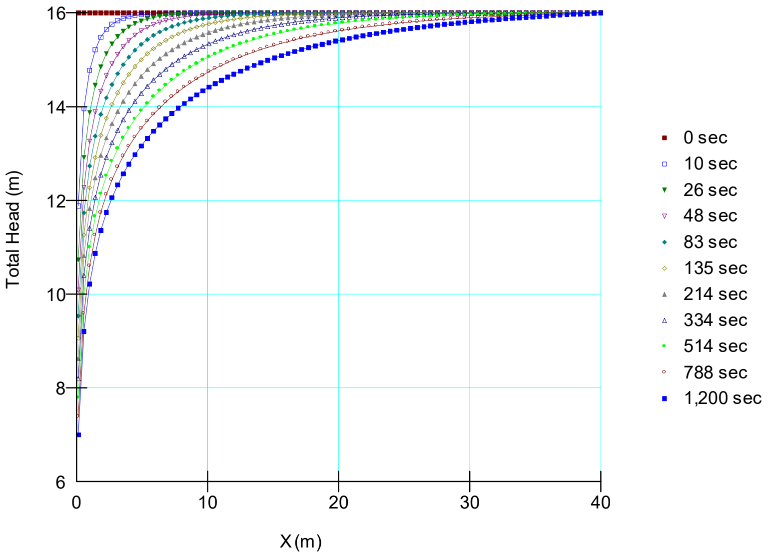


Figure 6. Drawdown curves with time.

Figure 7 compares the two solutions at a radial distance of 4 m from the well central axis. The agreement between the two solutions is remarkably good. The two solutions overlap in early time steps, but begin to diverge as the duration of the pumping increases. The two solutions, however, remain very similar even after 20 minutes of pumping has occurred.

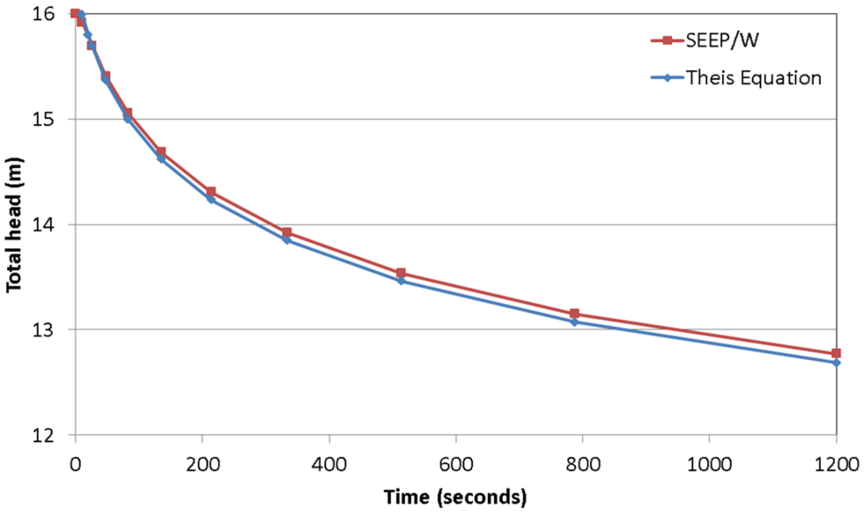


Figure 7. Comparison of the SEEP/W solution with the closed-form Theis solution.

Care must be taken when conducting this type of modeling to ensure that the duration is not too long. Once the drawdown is able to reach the far-field boundary, the results become rather meaningless. Specifying a total head at the far-field boundary infers that there is sufficient water available to maintain the head, which is usually not the actual case in the field. If it is desirable to run the analysis past this time, the mesh and domain should be extended further.

Another way to check that the analysis has not been run for too long of a duration is to use the Thiem steady-state solution:

$$h_0 - h = s = \frac{Q}{2\pi T} ln\left(\frac{r}{R}\right)$$

Equation 7

For the example here, the radius of influence (R) is 40 m. Therefore, the steady-state drawdown at a radial distance from the well (r) of 4 m is 4.3 m. In other words, the analysis should not be run longer than when the drawdown at a 4 m radial distance is approaching 4.3 m.

The specified pump rate is approximately 0.118 m^3/sec. So after 20 minutes of pumping, the total volume of water leaving the domain is approximately 141.37 m^3. The resulting cumulative water volume leaving the domain in the SEEP/W simulation is shown in Figure 7, where the last data point of 20 minutes (or 1200 seconds) is also 141.37 m^3. To create this graph, the “use entire domain” was chosen as the subdomain, with the nodal locations chosen along the geometry line representing the well screen. The negative sign in this graph indicates that the water is leaving the model domain.

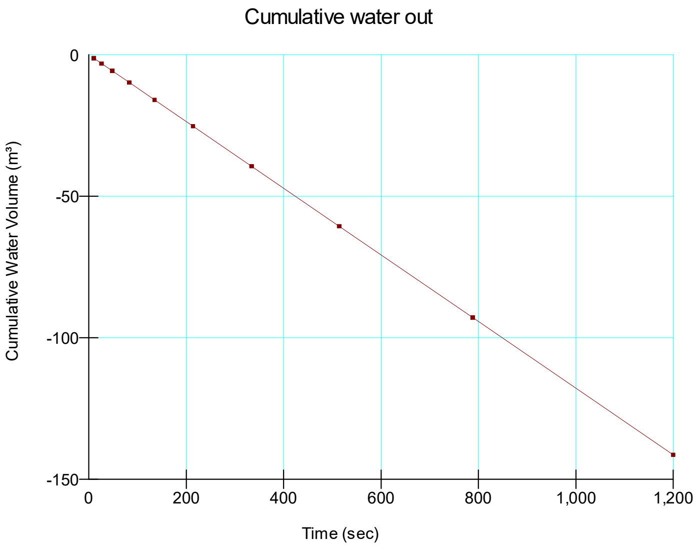


Figure 8. Cumulative water volume pumped from the aquifer.

Summary and Conclusions

The objective of this example was to illustrate the ability of SEEP/W to accurately predict the drawdown of a confined aquifer to pumping. The resulting drawdown was compared with the Theis solution to ensure that an accurate representation was simulated.

This is type of analysis is sensitive to the compressibility of the aquifer. In SEEP/W terminology, the analysis is sensitive to m_v, which represents the coefficient of volume compressibility. If the aquifer is very stiff (low compressibility), the water will drain very fast, as the aquifer does not have much storage. It is good modeling practice to experiment with a range of m_v values to gain a good understanding of how the compressibility affects the performance of the aquifer.

References

Schwartz, F.W. and Zhang, H. 2003. Fundamentals of Ground Water. John Wiley & Sons, Inc. New York, NY. pp. 220-226.